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New Discrete-Time, Fuzzy-Sliding-Mode Control with Application to Smart Structures

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A new discrete-time, fuzzy-sliding-mode controller with application to vibration control of a smart structure featuring a piezofilm actuator is presented. First, a discrete-time model with mismatched uncertainties is considered for the design of a discrete-time, sliding-mode controller, which consists of an equivalent part and a discontinuous part. In the design of the equivalent part, an equivalent controller separation method is adopted to achieve faster convergence to a sliding surface without extension of a sliding region, in which a system robustness cannot be guaranteed. The discontinuous control part is then constructed on the basis of the sliding and convergence conditions. Subsequently, the discrete-time, fuzzy-sliding-mode controller is formulated by employing a fuzzy technique to appropriately determine principal control parameters such as a discontinuous feedback gain. The controller is then experimentally realized, and control results of transient, forced, and random vibrations of the smart structure are presented to demonstrate the effectiveness of the proposed method.

Introduction

RECENTLY, with the aid of advanced technologies in computer and material sciences, advanced lightweight structural systems have been achieved in various research fields such as robotics, space structures, and manufacturing. Especially, the emergence of so-called smart materials has accelerated successful development of the advanced structural systems.[1,2] Thus far, the smart materials include electrorheological fluids, shape memory alloys, piezoelectric materials, and optical fibers.

In practice, most flexible structures using the smart materials can be easily subjected to parameter variations, such as natural frequency and external disturbances.[3,4] Hence, these systems require robust control algorithms such as sliding-mode control (SMC) rather than the conventional proportional–integral–derivative controller. Sliding modes that are the principal modes in the variable structure system are obtained by appropriate discontinuous control laws.[5,6] In the sliding mode, the control systems have robust properties to the parameter variations and external disturbances. The sliding motion is independent on the controller and is determined solely by the properties of the control plant and the positions of sliding surfaces. This allows the initial control problem to be decoupled into two independent lower-dimensional subproblems; the controller is activated only for creating the sliding mode, whereas required characteristics of the motion over the intersection of the sliding surfaces are to be provided by a suitable choice of their equations.[7–9]

In general, the SMC has been designed based on the continuous-time system. Its implementation by a digital computer, however, requires certain sampling processes. It may be impossible that the sampling processes are exactly realized in the continuous-time SMC (CSMC) system. In other words, because the sampling processes bring not only unwanted chattering of the control input along the predesigned sliding surfaces but also possible instability of the control system with an unnecessary large gain, practical implementation of the CSMC is sometimes very difficult. This leads to the study of the discrete-time SMC (DSMC) system.[10–13]

The main discrepancy between the DSMC and the CSMC is the determination of the existence condition of the sliding mode. Sarpturk et al.[10] have presented the existence condition to determine a stable sliding-mode controller for the discrete-time system. This condition requires a controller gain to have upper and lower bounds. But if the system has uncertainties, there exists a region in the neighborhood of the sliding surface where the feedback gain cannot be defined. This region is called a sliding region. Furuta[11] and Wang et al.[12] used an equivalent controller to reduce the sliding region. However, this method makes the control system very sensitive to the uncertainties inside the sliding region. Furuta and Pan[13] have suggested a β-equivalent controller to attenuate the sensitivity. However, this solution makes the sliding region enlarge. This may cause adverse chattering in the vibration control of flexible structures.

In this study, we propose a new discrete-time, fuzzy-sliding-mode controller to remove these drawbacks and apply it to a robust vibration control of a smart flexible structure with mismatched system parameter variations and/or external disturbances. After designing a stable sliding surface using the eigenvalue assignment method, the β-equivalent controller is constructed. Inside the sliding region, only this controller is to be activated to get faster convergence to the sliding surface without extension of the sliding region. A discontinuous controller is then designed on the basis of the sliding and convergence conditions. This controller is to be activated only outside the sliding region to increase the robustness to system uncertainties. Control parameters of the discontinuous feedback gain and β are dependent on the positions of representative points RP and the predetermined sliding surface. Therefore, it is, in general, difficult to select proper control parameters. In this study, a discrete-time, fuzzy-sliding-mode controller (DFSMC) is formulated by incorporating a fuzzy technique to determine appropriate control parameters. The DFSMC is then experimentally realized for the vibration control of the smart structure featuring the piezofilm actuator. Experimental results of transient, forced, and random vibrations are presented to demonstrate excellent control performance and robustness of the proposed controller.

DSMC

We consider a single-input, uncertain-discrete-time system given by

$$\boldsymbol{x}(k+1) = (\Phi_0 + \Delta\Phi)\boldsymbol{x}(k) + (\Gamma_0 + \Delta\Gamma)u(k) + \boldsymbol{d}(k) \tag{1}$$

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where

$$\Phi_0 = \begin{bmatrix} \phi_{11,0} & \cdots & \phi_{1n,0} \\ \vdots & & \\ \phi_{n1,0} & \cdots & \phi_{nn,0} \end{bmatrix}, \qquad \Gamma_0 = \begin{bmatrix} \gamma_{1,0} \\ \vdots \\ \gamma_{n,0} \end{bmatrix}$$

$$\phi_{ij,0} = \frac{\phi_{ij,\min} + \phi_{ij,\max}}{2}, \qquad \gamma_{i,0} = \frac{\gamma_{i,\min} + \gamma_{i,\max}}{2}$$

$$i, j = 1, \ldots, n$$

$$\Delta\Phi = \begin{bmatrix} \delta\phi_{11}(k) & \cdots & \delta\phi_{1n}(k) \\ \vdots & & \\ \delta\phi_{n1}(k) & \cdots & \delta\phi_{nn}(k) \end{bmatrix}$$

$$\Delta\Gamma = \begin{bmatrix} \delta\gamma_1(k) \\ \vdots \\ \delta\gamma_n(k) \end{bmatrix}, \qquad \boldsymbol{d}(k) = \begin{bmatrix} d_1(k) \\ \vdots \\ d_n(k) \end{bmatrix}$$

where Φ_0 and Γ_0 are the nominal system matrix and nominal input vector, respectively; $\Delta\Phi$ and $\Delta\Gamma$ are corresponding uncertain parts; and $\boldsymbol{d}(k)$ is the external disturbance including excitation force. Here, $\phi_{ij,\min}$ and $\phi_{ij,\max}$ are the lower and upper bounds of the corresponding element $\phi_{ij,0}$. The matching condition[14] for system (1) is available when certain functions $\boldsymbol{D}_1(k)$, $D_2(k)$, and $D_3(k)$ exist to satisfy $\Delta\Phi(k) = \Gamma_0\boldsymbol{D}_1(k)$, $\Delta\Gamma(k) = \Gamma_0 D_2(k)$, and $\boldsymbol{d}(k) = \Gamma_0 D_3(k)$. Obviously, the proposed system (1) does not satisfy the matching condition for the imposed uncertainties. However, for the formulation of the controller $u(k)$, the maximum parameter variations and disturbance need to be defined as follows:

$$\begin{aligned} \bar{\phi}_{ij} &\equiv \max_k[|\delta\phi_{ij}(k)|] = \phi_{ij,\max} - \phi_{ij,0} = (\phi_{ij,\max} - \phi_{ij,\min})/2 \\ \bar{\gamma}_i &\equiv \max_k[|\delta\gamma_i(k)|] = \gamma_{i,\max} - \gamma_{i,0} = (\gamma_{i,\max} - \gamma_{i,\min})/2 \qquad (2) \\ \bar{d}_i &\equiv \max_k[|d_i(k)|], \qquad i = 1, \ldots, n \end{aligned}$$

Equivalent Controller

In this study, the control objective is to enforce the vibration of the smart structure to zero, a regulating control problem. Thus, for the system (1), we may set a sliding surface in the state space as follows:

$$s(k) = \boldsymbol{C}\boldsymbol{x}(k) \tag{3}$$

where $\boldsymbol{C} = [c_1 \cdots c_n]$ is the surface gradient vector. The surface gradient vector should be determined so that the sliding surface (3) itself is stable in the state space. This can be accomplished by assigning desired eigenvalues for the nominal system matrix Φ_0 in Eq. (1) (Ref. 6). Now, the equivalent controller for the nominal system is obtained such that the state variables rest on the surface of $s(k+1) = s(k)$ for all k given by

$$\boldsymbol{C}[\Phi_0\boldsymbol{x}(k) + \Gamma_0 u(k)] = \boldsymbol{C}\boldsymbol{x}(k)$$

Thus,

$$u_{\mathrm{eq}}(k) = \boldsymbol{F}_{\mathrm{eq}}\boldsymbol{x}(k) = -(\boldsymbol{C}\Gamma_0)^{-1}(\boldsymbol{C}\Phi_0 - \boldsymbol{C})\boldsymbol{x}(k) \tag{4}$$

In the sliding mode, the motion of the system is subjected to the following equations:

$$\boldsymbol{x}(k+1) = (\Phi_0 + \Gamma_0\boldsymbol{F}_{\mathrm{eq}})\boldsymbol{x}(k) = \Phi_{\mathrm{eq}}\boldsymbol{x}(k), \qquad s(k) = \boldsymbol{C}\boldsymbol{x}(k) = 0 \tag{5}$$

Here, one eigenvalue of Φ_{eq} is 1 and the others $(n-1)$ become predetermined desired eigenvalues (less than 1) (Ref. 11). Consequently, the nominal system with the equivalent controller (4) is marginally stable in the sliding mode.

On the other hand, the β-equivalent controller is determined from the relation of $s(k+1) = \beta s(k)$ as follows:

$$\boldsymbol{C}[\Phi_0\boldsymbol{x}(k) + \Gamma_0 u(k)] = \beta\boldsymbol{C}\boldsymbol{x}(k), \qquad 0 \le \beta < 1$$

This yields

$$u_{\mathrm{eq},\beta}(k) = \boldsymbol{F}_{\mathrm{eq},\beta}\boldsymbol{x}(k) = -(\boldsymbol{C}\Gamma_0)^{-1}(\boldsymbol{C}\Phi_0 - \beta\boldsymbol{C})\boldsymbol{x}(k) \tag{6}$$

The nominal system with the β-equivalent controller (6) is asymptotically stable inside the sliding region because β itself is one of the eigenvalues and it is less than 1 (Ref. 13).

As stated earlier, using the equivalent controller (4), we can reduce the sliding region, but the system sensitivity to the uncertainties increases. On the other hand, using the β-equivalent controller (6), we can attenuate the sensitivity with a tradeoff of the large sliding region. To keep only the advantage of each equivalent controller, in this study we propose an equivalent controller separation method; inside the sliding region, we activate only the β-equivalent controller (6), whereas the controller (4) is activated outside the sliding region by incorporating the discontinuous controller to be designed in the subsequent section. This switching activation does not cause instability of the system because the sliding mode dynamics with Eq. (4) or (6) is stable.

Discontinuous Controller

For the discrete-time system, the controller should be chosen to satisfy the following condition:

$$|s(k+1)| < |s(k)| \tag{7}$$

This condition may be decomposed into the following two inequalities:

$$[s(k+1) - s(k)]\,\mathrm{sgn}[s(k)] < 0 \tag{8a}$$

$$[s(k+1) + s(k)]\,\mathrm{sgn}[s(k)] > 0 \tag{8b}$$

Sarpturk et al.[10] called the inequalities (8a) and (8b) the sliding condition and convergence condition, respectively.

Let the controller in the uncertain system (1) be

$$u(k) = u_{\mathrm{eq}}(k) + u_d(k) \tag{9}$$

where $u_{\mathrm{eq}}(k)$ is the equivalent controller given by Eq. (4) and $u_d(k)$ is the discontinuous controller to be designed. Before designing the discontinuous controller, we assume that $\boldsymbol{C}(\Gamma_0 + \Delta\Gamma)$ is nonsingular for all k. This assumption can be then expressed by[12]

$$\left|\sum_{i=1}^{n} c_i\gamma_{i,0}\right| > \sum_{i=1}^{n} |c_i\bar{\gamma}_i| \tag{10}$$

Now, with the condition (10), the discontinuous controller for the uncertain system (1) can be determined by the following theorem.

Theorem: The discrete-time, sliding-mode controller (9) makes the state trajectory of the system (1) converge robustly to the inside of the sliding region from the outside, if the equivalent controller is selected as Eq. (4) and the discontinuous controller is chosen by

$$u_d(k) = -h(k)\,\mathrm{sgn}[\boldsymbol{C}\Gamma_0 s(k)]\sum_{i=1}^{n}|x_i(k)| \tag{11}$$

where, outside the sliding region, $h_s(k) < h(k) < h_c(k)$ and, inside the sliding region, $h(k) = 0$:

$$h_s(k) = \frac{\sup[H_2(k)]}{\inf[H_1(k)]}, \qquad h_c(k) = \frac{2|s(k)| - \sup[H_2(k)]}{\sup[H_1(k)]}$$

and $\inf[H_1(k)]$, $\sup[H_1(k)]$, and $\sup[H_2(k)]$ are defined by

$$\begin{aligned} \inf[H_1(k)] &\equiv \inf\left[|\boldsymbol{C}(\Gamma_0 + \Delta\Gamma)|\sum_{i=1}^{n}|x_i(k)|\right] \\ &= \inf(|\boldsymbol{C}(\Gamma_0 + \Delta\Gamma)|)\sum_{i=1}^{n}|x_i(k)| \\ &= \left(\left|\sum_{i=1}^{n} c_i\gamma_{i,0}\right| - \sum_{i=1}^{n}|c_i\bar{\gamma}_i|\right)\sum_{i=1}^{n}|x_i(k)| \end{aligned}$$

$$\sup[H_1(k)] \equiv \sup\left[|\boldsymbol{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})| \sum_{i=1}^{n} |x_i(k)|\right]$$

$$= \sup[|\boldsymbol{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})|] \sum_{i=1}^{n} |x_i(k)|$$

$$= \left(\left|\sum_{i=1}^{n} c_i \gamma_{i,0}\right| + \sum_{i=1}^{n} |c_i \bar{\gamma}_i|\right) \sum_{i=1}^{n} |x_i(k)|$$

$$\sup[H_2(k)] \equiv \sup\{\boldsymbol{C}[\Delta\boldsymbol{\Phi}\boldsymbol{x}(k) + \Delta\boldsymbol{\Gamma} u_{\mathrm{eq}} + \boldsymbol{d}(k)]\}$$

$$= \sup\left[\sum_{i=1}^{n}\sum_{j=1}^{n} c_i \delta\phi_{ij}(k) x_j(k) + \sum_{i=1}^{n}\sum_{j=1}^{n} c_i \delta\gamma_i(k) u_{\mathrm{eq}} + \sum_{i=1}^{n} c_i d_i(k)\right]$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} |c_i \bar{\phi}_{ij} x_j(k)| + |u_{\mathrm{eq}}(k)| \sum_{i=1}^{n} |c_i \bar{\gamma}_i| + \sum_{i=1}^{n} |c_i \bar{d}_i|$$

Proof: From the system equation (1) and the sliding surface equation (3), we can get $s(k+1)$ as follows:

$$s(k+1) = \boldsymbol{C}(\boldsymbol{\Phi}_0 + \Delta\boldsymbol{\Phi})\boldsymbol{x}(k) + \boldsymbol{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})u(k) + \boldsymbol{C}\boldsymbol{d}(k)$$
$$= \boldsymbol{C}[\Delta\boldsymbol{\Phi}\boldsymbol{x}(k) + \Delta\boldsymbol{\Gamma} u_{\mathrm{eq}}(k) + \boldsymbol{d}(k)] + \boldsymbol{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})u_d(k) + s(k) \qquad (12)$$

Substituting Eq. (12) and the discontinuous controller (11) into the sliding condition (8a) and the convergence condition (8b), respectively, yields

$$h(k)|\boldsymbol{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})| \sum_{i=1}^{n} |x_i(k)| > \boldsymbol{C}[\Delta\boldsymbol{\Phi}\boldsymbol{x}(k) + \Delta\boldsymbol{\Gamma} u_{\mathrm{eq}}(k) + \boldsymbol{d}(k)]\,\mathrm{sgn}[s(k)] \qquad (13a)$$

$$h(k)|\boldsymbol{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})| \sum_{i=1}^{n} |x_i(k)| < 2|s(k)| + \boldsymbol{C}[\Delta\boldsymbol{\Phi}\boldsymbol{x}(k) + \Delta\boldsymbol{\Gamma} u_{\mathrm{eq}}(k) + \boldsymbol{d}(k)]\,\mathrm{sgn}[s(k)] \qquad (13b)$$

From these inequalities, we can get the lower bound $h_s(k)$ and the upper bound $h_c(k)$ of the discontinuous gain $h(k)$, which are given in the theorem. The $h_s(k)$ and $h_c(k)$ are called sliding gain and convergence gain, respectively. From the fulfillment of the sliding condition (8a) and the convergence condition (8b), the convergence of the state trajectory to the inside of the sliding region is guaranteed. This briefly concludes the proof.

DFSMC

Inside the sliding region, the value of β in Eq. (6) is physically relative to the control gain. The β is closer to zero; the control gain is larger. If we choose sufficiently small β, we may achieve fast and robust control effect. But this may cause unwanted chattering in the vibration control. Outside the sliding region, the discontinuous gain $h(k)$ in Eq. (11) has characteristics similar to β. Therefore, proper choice of β and $h(k)$ is very important to achieve excellent control performance. However, it is, in general, difficult to select proper β and $h(k)$ because they are time varying. In this study, we adopt a fuzzy technique to take account of this problem.

The fuzzy control theory has been applied to the control of electrical systems, robotic systems, and various mechanical systems.[15] The prominent advantage of the fuzzy controller (FC) is that it can effectively control complex, ill-defined systems having nonlinearities, parameter variations, and disturbances. When we design an FC, how to select appropriate FC rules is the first problem to be resolved. For this, we consider the upper bound $h_c(k)$ and the lower bound $h_s(k)$ of the discontinuous feedback gain. These bounds, $h_c(k)$ and $h_s(k)$, are equal to the boundary of the sliding region. Thus, we can obtain the information of this boundary and define the following function:

$$s_{\mathrm{bound}}(k) \equiv \frac{h_s(k)\cdot \sup[H_1(k)] + \sup[H_2(k)]}{2} \qquad (14)$$

We choose the control logic as the following condition statements.

1) Inside the sliding region:

IF RP is far from the sliding surface,
THEN β is small and vice versa. (15)

2) Outside the sliding region:

IF RP is far from the sliding surface,
THEN $h(k)$ is large and vice versa. (16)

For the construction of the FC that has linguistic rules, we choose two fuzzy variables for β and $h(k)$ as follows.

1) Inside the sliding region:

$$\sigma_\beta = \frac{|s(k)|}{s_{\mathrm{bound}}(k)}, \qquad \Delta\sigma_\beta = \sigma_\beta(k) - \sigma_\beta(k-1) \qquad (17)$$

2) Outside the sliding region:

$$\sigma_h = \frac{s_{\mathrm{bound}}(k)}{|s(k)|}, \qquad \Delta\sigma_h = \sigma_h(k) - \sigma_h(k-1) \qquad (18)$$

Now, two linguistic input variables are defined to describe these fuzzy variables as follows:

$$\tilde{\sigma} = \{\mathrm{VS, SM, ME, LA, VL}\}, \qquad \Delta\tilde{\sigma} = \{\mathrm{NL, NS, ZO, PS, PL}\} \qquad (19)$$

where VS is very small, SM small, ME medium, LA large, VL very large, NL negatively large, NS negatively small, ZO zero, PS positively small, and PL positively large. Also, linguistic output variables are defined to describe the β and $h(k)$ as follows:

$$\tilde{\beta} = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5\}, \qquad \tilde{h} = \{h_1, h_2, h_3, h_4, h_5\} \qquad (20)$$

where β_i, h_i $(i = 1, \ldots, 5)$ are fuzzy values of β and $h(k)$, respectively. The input–output relations of the FC with fuzzy variables (19) and (20) inside and outside the sliding region, respectively, are

$$\tilde{\sigma}, \Delta\tilde{\sigma} \to \tilde{\beta}, \qquad \tilde{\sigma}, \Delta\tilde{\sigma} \to \tilde{h} \qquad (21)$$

Table 1 is a lookup table of fuzzy control rules adopted in the present study, i.e.,

$R_{\beta,11}$: IF $\tilde{\sigma}$ is VL and $\Delta\tilde{\sigma}$ is NL inside the sliding region,
THEN $\tilde{\beta}$ is β_1
⋮ (22)
$R_{\beta,55}$: IF $\tilde{\sigma}$ is VS and $\Delta\tilde{\sigma}$ is PL inside the sliding region,
THEN $\tilde{\beta}$ is β_5

$R_{h,11}$: IF $\tilde{\sigma}$ is VL and $\Delta\tilde{\sigma}$ is NL outside the sliding region,
THEN $\tilde{h}$ is h_2
⋮ (23)
$R_{h,55}$: IF $\tilde{\sigma}$ is VS and $\Delta\tilde{\sigma}$ is PL outside the sliding region,
THEN $\tilde{h}$ is h_3

This fuzzy algorithm can be inferred from the center-of-gravity method.[16]

Table 1 Linguistic fuzzy rule base for the $\beta[h(k)]$

$\Delta\tilde{\sigma}$ \ $\tilde{\sigma}$	VL	LA	ME	SM	VS
NL	$\beta_1(h_2)$	$\beta_1(h_3)$	$\beta_2(h_3)$	$\beta_3(h_4)$	$\beta_3(h_5)$
NS	$\beta_1(h_2)$	$\beta_2(h_2)$	$\beta_2(h_3)$	$\beta_3(h_4)$	$\beta_3(h_5)$
ZO	$\beta_2(h_2)$	$\beta_2(h_2)$	$\beta_3(h_3)$	$\beta_3(h_3)$	$\beta_4(h_4)$
PS	$\beta_2(h_1)$	$\beta_2(h_2)$	$\beta_3(h_2)$	$\beta_4(h_3)$	$\beta_4(h_4)$
PL	$\beta_2(h_1)$	$\beta_3(h_1)$	$\beta_4(h_2)$	$\beta_4(h_3)$	$\beta_5(h_3)$

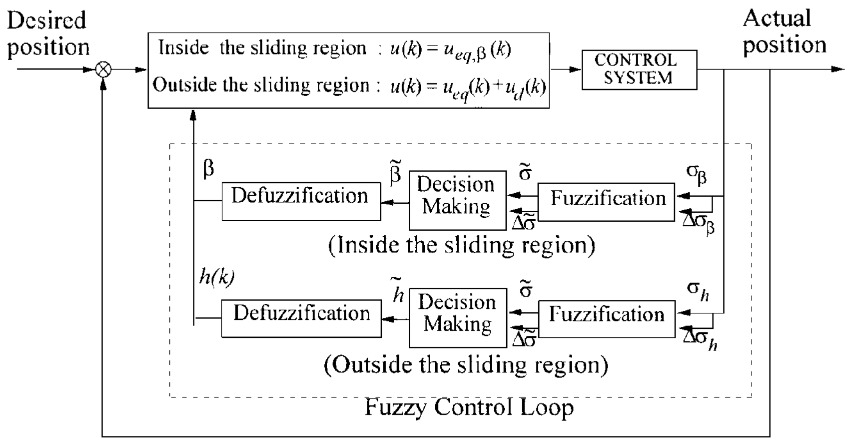


Fig. 1 Block diagram of the proposed DFSMC.

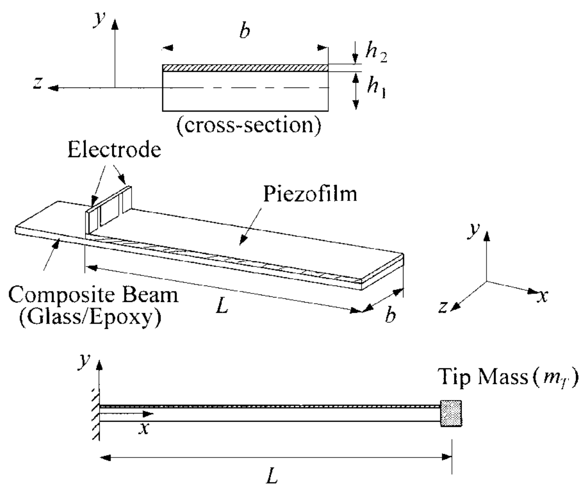


Fig. 2 Schematic diagram of the smart structure.

Figure 1 presents the block diagram of the proposed DFSMC. The basic configuration of the FC consists of three components: a fuzzification interface, a decision-making logic, and a defuzzification interface. In the fuzzification interface, σ_β, $\Delta\sigma_\beta$ or σ_h, $\Delta\sigma_h$ are estimated, and these are modified into linguistic values $\tilde{\sigma}$ and $\Delta\tilde{\sigma}$. The decision-making logic is the kernel of the FC inasmuch as it has the capability of both simulating decision making based on fuzzy concepts and inferring fuzzy control actions. In the defuzzification interface, fuzzy output β or $h(k)$ decided from the decision making logic is changed into a numerical value for the purpose of using real control input. Hence, the β-equivalent controller and the discontinuous controller can be formulated as follows:

$$u_{\text{eq},\beta}(k) = -(\boldsymbol{C\Gamma}_0)^{-1}[\boldsymbol{C\Phi}_0 - \beta(\tilde{\sigma}, \Delta\tilde{\sigma})\boldsymbol{C}]\boldsymbol{x}(k) \tag{24}$$

$$u_d(k) = -h(\tilde{\sigma}, \Delta\tilde{\sigma})\,\text{sgn}[\boldsymbol{C\Gamma}_0 s(k)] \sum_{i=1}^{n} |x_i(k)| \tag{25}$$

It is evident from Fig. 1 that only the β-equivalent controller (24) is activated inside the sliding region, whereas the controllers (4) and (25) are activated outside the sliding region.

Application to a Smart Structure

System Modeling

Consider a smart flexible cantilevered beam (as shown in Fig. 2) in which the piezofilm (PF), also known as a polyvinylidenefluoride (PVDF), is perfectly bonded on the upper surface of a host material structure (composite beam) as an actuator. Upon applying a voltage $V(t)$ to the PF, a strain is produced in the PF layer, and the resulting bending moment in the beam is obtained as follows[17]:

$$M = -d_{31} \cdot \frac{h_1 + h_2}{2} \cdot \frac{E_1 E_2 h_1 b}{E_1 h_1 + E_2 h_2} \cdot V(t) = c \cdot V(t) \tag{26}$$

where c is a constant dependent on mechanical properties and the geometry of the host material and the PF. E_1 and E_2 are the Young's modulus of the composite beam and the PF, respectively; and d_{31} denotes a piezoelectric strain constant. Applying energy equations and Hamilton's principle, the governing equation of motion is obtained. Subsequently, upon retaining a finite number of control modes, a reduced-order dynamic model is achieved in the state-space representation as follows[18]:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t) + \boldsymbol{f}(t), \qquad y(t) = \boldsymbol{E}\boldsymbol{x}(t) \tag{27}$$

where

$$\boldsymbol{x} = [q_1 \quad \dot{q}_1 \quad q_2 \quad \dot{q}_2 \quad \cdots \quad q_m \quad \dot{q}_m]^T = [x_1 \quad x_2 \quad \cdots \quad x_{2m}]^T$$

$$u(t) = V(t)$$

$$\boldsymbol{f}(t) = [0 \quad f_1(t) \quad 0 \quad f_2(t) \quad \cdots \quad 0 \quad f_m(t)]^T$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & & & \\ -\omega_1^2 & -2\zeta_1\omega_1 & & \boldsymbol{0} & \\ & & \ddots & & \\ & \boldsymbol{0} & & 0 & 1 \\ & & & -\omega_m^2 & -2\zeta_m\omega_m \end{bmatrix}$$

$$\boldsymbol{B} = -\frac{c}{I_t} \times \left[0 \quad \int_0^L \frac{\partial^2\phi_1}{\partial x^2}\,\mathrm{d}x \quad 0 \quad \int_0^L \frac{\partial^2\phi_2}{\partial x^2}\,\mathrm{d}x \quad \cdots \quad 0 \quad \int_0^L \frac{\partial^2\phi_m}{\partial x^2}\,\mathrm{d}x\right]^T$$

$$\boldsymbol{E} = [\phi_1(L) \quad 0 \quad \phi_2(L) \quad 0 \quad \cdots \quad \phi_m(L) \quad 0]$$

In Eq. (27), $f_i(t)$ is an unknown but bounded external force to disturb the ith mode. The ω_i and ζ_i denote the natural frequency and the damping ratio of the ith mode, respectively; and I_t is the generalized mass. Note that the output matrix $\boldsymbol{E}$ is related to the tip position sensor.

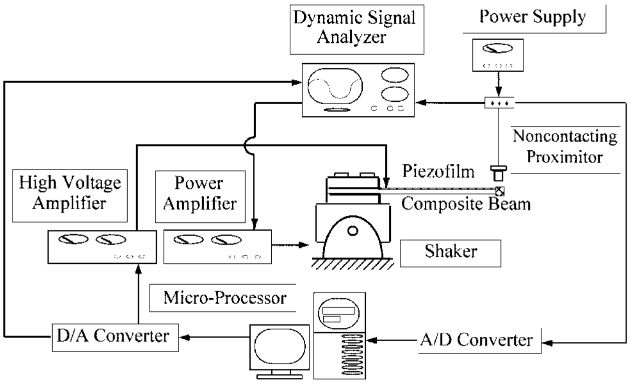


Fig. 3 Experimental apparatus for vibration control.

In practice, due to the variable tip mass, errors on measurement, and so on, possible variations of the parameters ω_i and ζ_i will occur, and these can be expressed as follows:

$$\omega_i(t) = \omega_{i,0} + \delta\omega_i(t), \qquad \zeta_i(t) = \zeta_{i,0} + \delta\zeta_i(t) \tag{28}$$

Now substituting Eq. (28) into Eq. (27) yields following uncertain state dynamic model:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= [\boldsymbol{A}_0 + \Delta\boldsymbol{A}(t)]\boldsymbol{x}(t) + \boldsymbol{B}u(t) + \boldsymbol{f}(t) \\ &= \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}u(t) + \boldsymbol{f}(t)\end{aligned} \tag{29}$$

Here we assume that all elements of the system and input matrices are piecewise smooth functions.

By using the zero-order-hold method,[19] the continuous-time system (29) can be written by the following discrete-time version:

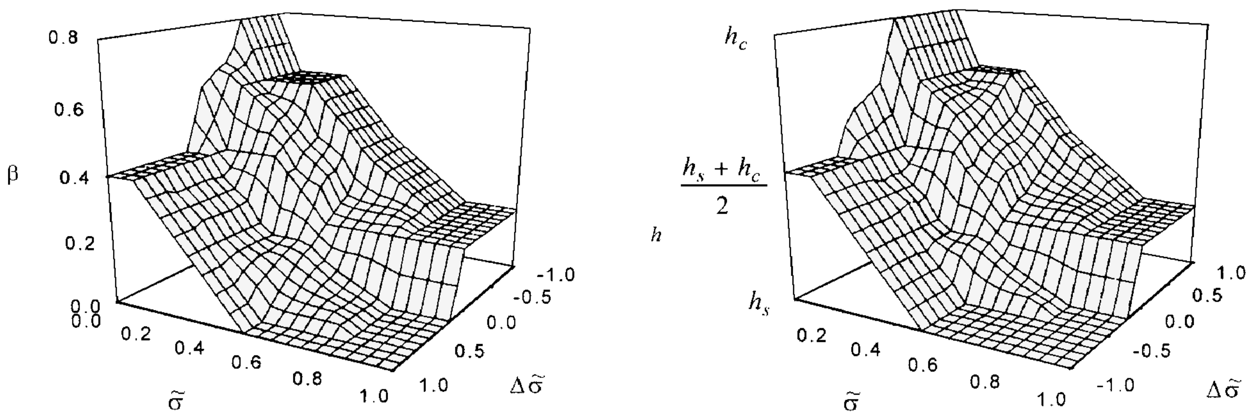
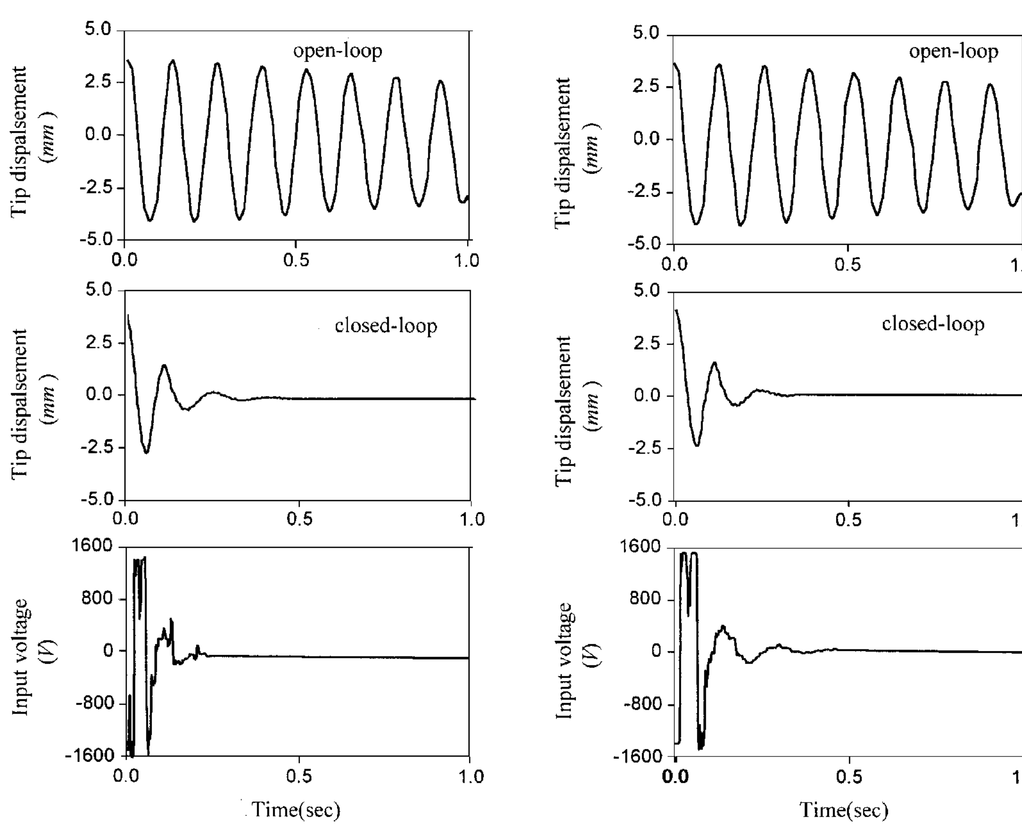


Fig. 4 Surfaces of the fuzzy output variables.



a) Without the separation method

b) With the separation method

Fig. 5 Transient vibration control responses with tip mass.

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)u(k) + d(k) \tag{30}$$

where

$$\Phi(k) = \exp\left[\int_{t_k}^{t_{k+1}} A(\tau)\,d\tau\right] \cong \exp\left[A\left(\frac{t_k + t_{k+1}}{2}\right)T\right]$$

$$\Gamma(k) = \int_{t_k}^{t_{k+1}} \exp\left[\int_{\tau}^{t_{k+1}} A(\alpha)\,d\alpha\right] B(\tau)\,d\tau$$

$$\cong \int_0^T \exp\left[A\left(\frac{t_k + t_{k+1}}{2}\right)\tau\right] d\tau B\left(\frac{t_k + t_{k+1}}{2}\right)$$

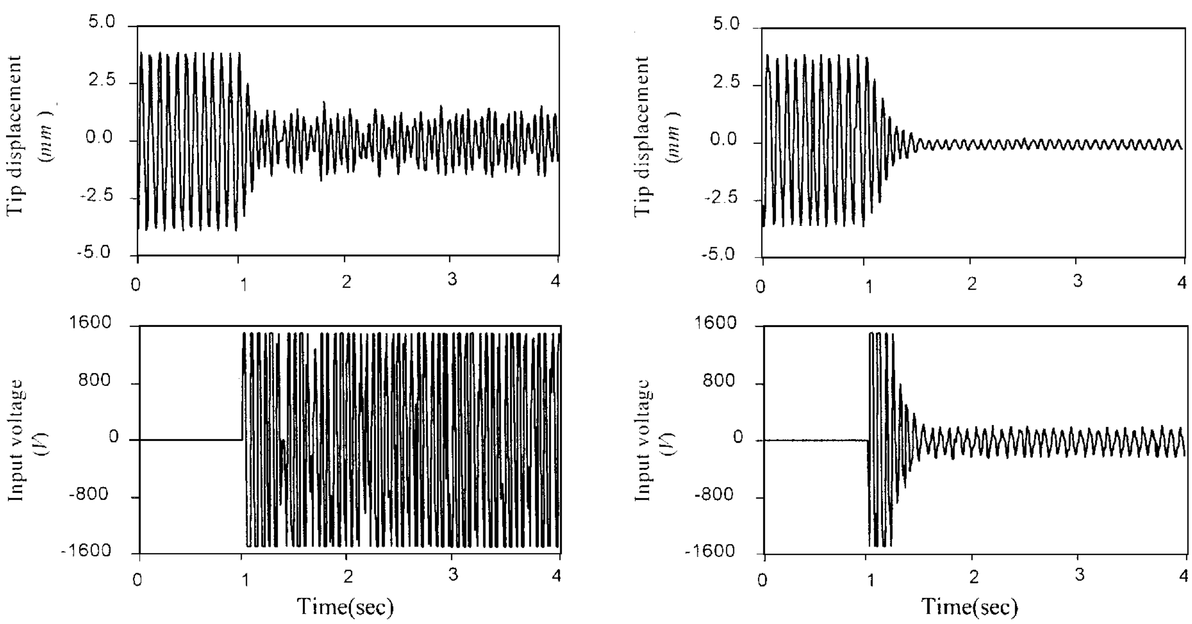
$$d(k) \cong \int_0^T \exp\left[A\left(\frac{t_k + t_{k+1}}{2}\right)\tau\right] f[(k+1)T - \tau]\,d\tau$$

where T is a sampling time and k is a sampling number. We can rewrite Eq. (30) as the form of Eq. (1) using the nominal part and the uncertain part as follows:

$$x(k+1) = (\Phi_0 + \Delta\Phi)x(k) + (\Gamma_0 + \Delta\Gamma)u(k) + d(k) \tag{31}$$

Control Results and Discussion

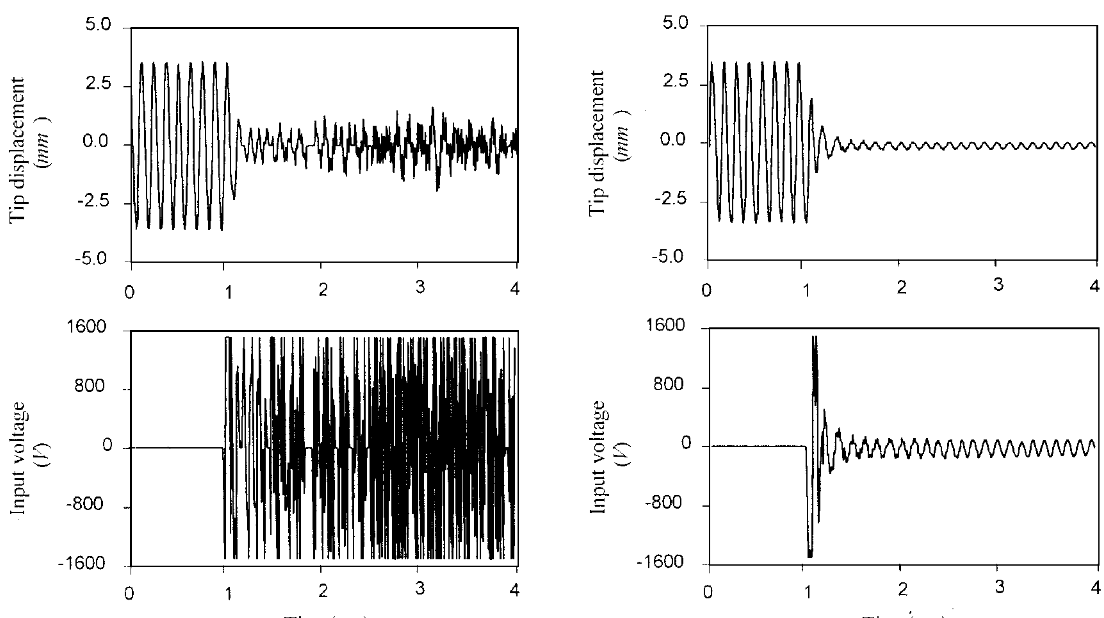
The dimensional and material specifications of the composite beam and the PF used in this study are given in Table 2. The system model parameters without tip mass were obtained from the experiment as follows: $\omega_1 = 11.8$ Hz, $\omega_2 = 75$ Hz, $\zeta_1 = 0.0072$, and $\zeta_2 = 0.0043$. The first and second flexible modes were considered as the primary modes to be controlled in the implementation. The position state variables were directly measured, and the velocity state variables were estimated using a reduced-order observer.[20] To demonstrate the robustness, the parameter variations were imposed in the controller design as follows: $\omega_1 = 11.8$ Hz $\pm$ 18%, $\omega_2 = 75$ Hz $\pm$ 18%, $\zeta_1 = 0.0072 \pm 10\%$, and $\zeta_2 = 0.0043 \pm 10\%$. The percentage of the variations was determined by considering



a) Without the separation method

b) With the separation method

Fig. 6 Forced vibration control responses without tip mass.



a) Without the separation method

b) With the separation method

Fig. 7 Forced vibration control responses with tip mass.

the change of the natural frequency and damping ratio due to the tip mass. The system was sampled with the sampling time of 0.01 s.

Figure 3 presents a schematic diagram of the experimental apparatus and associated instrumentation. The information of the tip displacement from a noncontacting displacement proximitor (Bently Nevada 7200) is fed back to the microprocessor through an A/D converter (DAS-20). Using the state variables, control input voltage is determined in the microprocessor (IBM PC 486) by means of the proposed DFSMC algorithm. The control voltage of the microprocessor is applied to the PF actuator through a D/A converter and a high-voltage amplifier (Trek 609A), which has a gain of 1000. The dynamic signal analyzer (Hewlett Packard 35665A) is used to record and analyze the experimental results.

Figure 4 presents the surfaces of the fuzzy output variables β and $h(k)$ obtained by the center-of-gravity method. We can easily see that this represents in nature the fuzzy control rules given by Table 1. Note that the membership functions of β and $h(k)$ are selected to be the same.

Table 2 Dimensional and mechanical properties of the composite beam and the PF

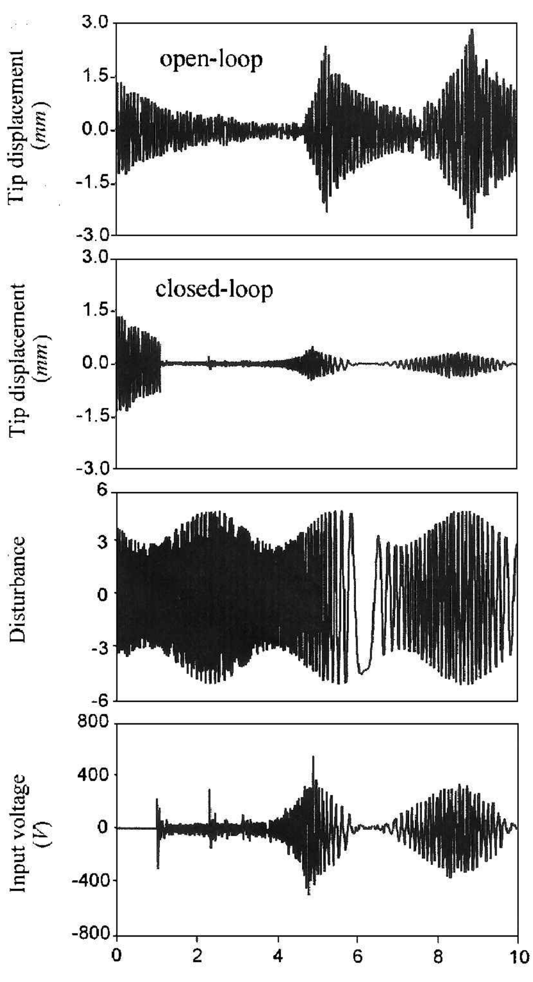
Young's modulus, GPa	Thickness, mm	Density, kg/m^3	Width, mm	Length, mm
Composite beam (glass/epoxy)				
6.4	0.65	1865	26.6	170
PF (PVDF)				
2	0.11	1780	26.6	170
Piezoelectric strain constant: 23×10^{-13} (m/m)/(V/m)				

Figure 5 presents measured transient vibration control responses, where initial tip displacement was imposed by 4.0 mm. The input control voltage was limited to ± 1.5 kV. From the open-loop response, we can see that the vibration decays only with the material damping. The closed-loop control response advocates that the imposed transient vibration was perfectly controlled by using the DFSMC. In the transient vibration control, however, the effectiveness of the equivalent controller separation method is not prominent.

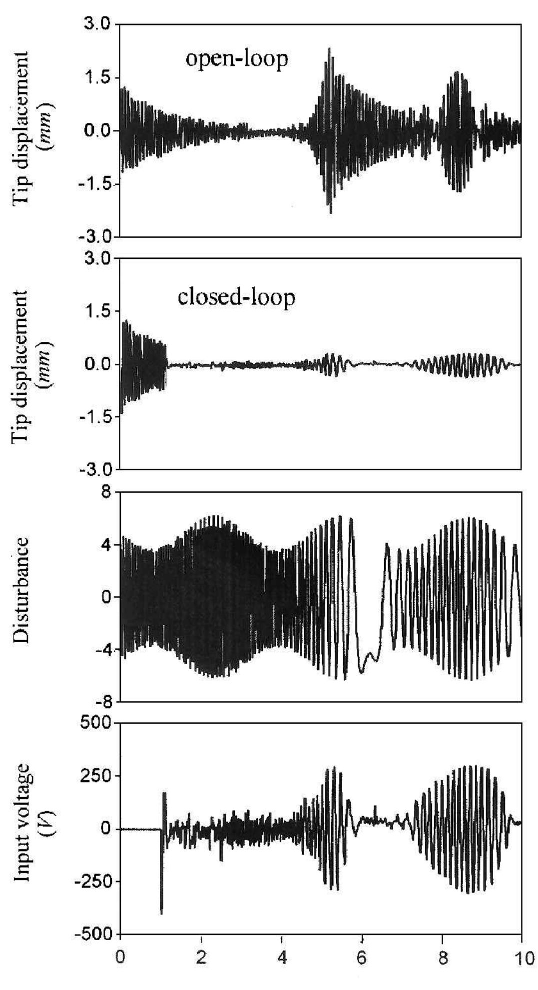
The measured control responses of the forced vibration without and with the tip mass are presented in Figs. 6 and 7, respectively. We can clearly observe that the proposed equivalent controller separation method considerably improves the system responses by attenuating the chattering of the control input as well as the tip displacement. The large chattering of the displacement without the separation method arises from the excessive supply of the control input voltage in the settling phase. This is because the sliding mode motion occurs on the sliding region, not inside the sliding region. Figure 8 shows measured random vibration control responses with the separation method. It is shown that the proposed controller provides good control performance against the disturbance (excitation), which has time-varying frequency and amplitude.

Conclusions

A new DFSMC was formulated to attenuate the chattering of the vibration and also to achieve robustness of the system uncertainties. In the design of the controller, an equivalent controller separation method was employed for reaching the sliding surface faster without increment of the sliding region. The controller was incorporated with a fuzzy technique to determine appropriate control parameters



a) Without tip mass



b) With tip mass

Fig. 8 Random vibration control responses with the separation method.

such as the discontinuous feedback gain. The controller has been successfully applied to the transient, forced, and random vibration control of a smart structure featuring a PF actuator. Finally, note that, without any modifications, the proposed control method can be applied to many other physical systems subjected to parameter variations and external disturbances.

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References

[1]Bailey, T., and Hubbard, J. E., "Distributed Piezoelectric-Polymer Active Vibration Control of a Cantilever Beam," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 5, 1985, pp. 605–611.

[2]Rogers, C. A., Baker, K. D., and Jaeger, C. A., "Introduction to Smart Materials and Structures," *Proceedings of ARO Workshop on Smart Materials, Structures and Mathematical Issues* (Blacksburg, VA), Technomic, Lancaster, PA, 1988, pp. 17–28.

[3]Rao, W. S., Butler, R., and Zhao, W., "Robust Multiobjective Controllers for Smart Structures," *Proceedings of the 5th International Conference on Adaptive Structures* (Sendai, Japan), Technomic, Lancaster, PA, 1995, pp. 163–172.

[4]Watanabe, T., and Toshida, K., "Robust Control for Flexible Structures Taking Account of Parameter Vibrations and Residual Dynamics," *Proceedings of the 5th International Conference on Adaptive Structures* (Sendai, Japan), Technomic, Lancaster, PA, 1995, pp. 387–396.

[5]Utkin, V. I., *Sliding Modes and Their Application in Variable Structure Systems*, MIR, Moscow, 1984, pp. 78–101.

[6]Chern, T. L., and Wu, Y. C., "Design of Integral Variable Structure Controller and Application to Electrohydraulic Velocity Servomechanism," *IEE Proceedings-D*, Vol. 138, No. 2, 1991, pp. 439–444.

[7]Choi, S. B., Park, D. W., and Jayasuriya, S., "A Time-Varying Sliding Surface for Fast and Robust Tracking Control of Second-Order Uncertain Systems," *Automatica*, Vol. 30, No. 5, 1994, pp. 899–904.

[8]Slotine, J. J. E., and Sastry, S. S., "Tracking Control of Non-Linear Systems Using Sliding Surfaces with Application to Robot Manipulators," *International Journal of Control*, Vol. 38, No. 2, 1983, pp. 465–492.

[9]Young, K. K. D., "Controller Design for a Manipulator Using Theory of Variable Structure Systems," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. SMC-8, No. 2, 1987, pp. 101–109.

[10]Sarpturk, S. Z., Istefanopulos, Y., and Kaynak, O., "On the Stability of Discrete-Time Sliding Mode Control Systems," *IEEE Transactions on Automatic Control*, Vol. 32, No. 2, 1983, pp. 930–932.

[11]Furuta, K., "Sliding Mode Control of a Discrete System," *Systems and Control Letters*, Vol. 14, No. 1, 1990, pp. 145–152.

[12]Wang, W. J., Wu, G. H., and Yang, D. C., "Variable Structure Control Design for Uncertain Discrete-Time Systems," *IEEE Transactions on Automatic Control*, Vol. 39, No. 1, 1994, pp. 99–102.

[13]Furuta, K., and Pan, Y., "Discrete-Time Adaptive VSS Control System Using Transfer Function," *IEEE Proceedings of the 32nd Conference on Decision and Control* (San Antonio, TX), Inst. of Electrical and Electronics Engineers, New York, 1993, pp. 1434–1438.

[14]Leitmann, G., "On the Efficacy of Nonlinear Control in Uncertain Linear Systems," *Journal of Dynamic Systems, Measurement and Control*, Vol. 102, June 1981, pp. 95–102.

[15]Lee, C. C., "Fuzzy Logic in Control Systems: Fuzzy Logic Controller—Parts I, II," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 20, No. 2, 1990, pp. 404–435.

[16]Li, Y. F., and Lau, C. C., "Development of Fuzzy Algorithms for Servo Systems," *IEEE Control Systems Magazine*, Vol. 9, No. 3, 1989, pp. 65–72.

[17]Choi, S. B., Cheong, C. C., and Kim, S. H., "Control of Flexible Structures by Distributed Piezofilm Actuators and Sensors," *Journal of Intelligent Material Systems and Structures*, Vol. 6, No. 5, 1995, pp. 430–435.

[18]Choi, S. B., Cheong, C. C., and Lee, C. H., "Position Tracking Control of a Smart Flexible Structure Featuring a Piezofilm Actuator," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 6, 1996, pp. 1364–1369.

[19]Franklin, G. F., Powell, J. D., and Workman, M. L., *Digital Control of Dynamic Systems*, Addison–Wesley, Reading, MA, 1990, pp. 102–121.

[20]Choi, S. B., Cheong, C. C., and Shin, H. C., "Sliding Mode Control of Vibration in a Single-Link Flexible Arm with Parameter Variations," *Journal of Sound and Vibration*, Vol. 179, No. 5, 1995, pp. 737–748.